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Transmission of electromagnetic waves through sub-wavelength channels

Jingjing Zhang^{1*}, Yu Luo², and Niels Asger Mortensen¹

¹ DTU Fotonik - Department of Photonics Engineering, Technical University of Denmark,
DK-2800 Kongens Lyngby, Denmark.

² Department of Physics, Blackett Laboratory, Imperial College London, London SW7 2AZ, UK.
*jinz@fotonik.dtu.dk

Abstract: We propose a method of tunneling electromagnetic (EM) waves through a channel with sub-wavelength cross section. By filling the channel with high- ϵ isotropic material and implementing two matching layers with uniaxial metamaterial substrates, the guided waves can go through the narrow channel without being cut off, as if it has just passed through the original empty waveguide. Both the magnitude and phase information of the EM fields can be effectively restored after passing this channel, regardless of the polarization of the incoming wave. The performance of this sub-wavelength channel, which is designed with coordinate transformation methodology, is studied theoretically and numerically.

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1. Introduction

Due to its potential applications in a wide range of fields, the transmission of electromagnetic energy through sub-wavelength channels has intrigued an extensive investigation in the recent years. The technology of exciting surface plasmon polaritons on both sides of the channel explores the possibility of overcoming the barrier of miniaturization and designing sub-wavelength waveguides [1]. Another inventive approach to the problem of squeezing electromagnetic energies through sub-wavelength channels was also introduced [2], making use of ϵ -near zero (ENZ) materials. This idea was later experimentally verified in microwave frequencies by two groups. One uses a planar waveguide setup, where enhanced transmission was measured when the narrow channel connecting waveguides was filled with patterned complementary split ring resonators [3]. The other achieved energy squeezing and tunneling through an ultra-narrow rectangular waveguide channel that mimicked zero-permittivity properties [4]. The scheme of utilizing ENZ materials to improve the energy transmission relies on the principle that there are no relevant reflection losses at abrupt junctions or bends where the permittivity of the materials is extremely small, and has the main advantage that the reflection reduction effect is independent of the shapes of the junctions or bends. However, this method cannot be used to guide EM fields through a channel where EM wave is inherently cut off. In addition, it is not effective for tunneling waves in high modes. In Ref [5], this method was improved when arrays of metallic wires were incorporated and was applied to transport an electromagnetic image through a tiny hole in the sub-wavelength spatial scale. In this paper, we suggest another possible solution to the problem of sub-wavelength tunneling, which is based on the coordinate transformation methodology. By exploiting a transformation which compresses the original space into a much narrower region, EM waves which are supposed to transmit in the original space will be confined in a channel whose cross-section is much smaller than the free-space wavelengths. This transformation gives rise to a set of materials parameters which could be achieved using natural positive materials as well as uniaxial metamaterials. Compared with the ENZ material approach in Ref [2], which only works for transverse electromagnetic (TEM) mode, the channel designed with transformation methodology can potentially squeeze and support all the incoming waves regardless of the polarizations and the transmission modes. Not only the magnitude but also the phase information of EM waves will be restored at the output interface of the channel. Both theoretical analysis and numerical simulations demonstrate the effectiveness of this sub-wavelength tunneling scheme.

2. Principle

Transformation optics offers the flexibility of controlling the paths of EM waves at will and has recently been widely used to design many EM devices with extraordinary capabilities, such as invisibility cloaks, field rotators, conformal antennas, highly directive antennas, beam expanders, etc [6–19]. Here we consider using the knowledge of the transformation to obtain the values of material parameters required to compress and guide the EM waves in a sub-wavelength channel. For simplification, we confine our discussion to the two-dimensional (2D) case, where the fields are invariant in z direction. As depicted in Fig. 1, a trapeziform region with the height h_2 , the short side $2l$, and the long side $2(l + d)$, has been compressed into the blue-gridded polygonal region. The upper and lower boundaries of the original and the transformed space are all perfect electric conducting (PEC) boundaries. The following equations describe such a transformation:

$$x' = x, y' = \frac{h_2 - h_1}{h_2} y + \frac{h_1}{d}(l + d + x), z' = z \quad \text{for } -(l + d) < x < -l, \quad (1-a)$$

$$x' = x, y' = \frac{h_2 - h_1}{h_2} y + \frac{h_1}{d}(l + d - x), z' = z \quad \text{for } l < x < l + d, \quad (1-b)$$

$$x' = x, y' = \frac{h_2 - h_1}{h_2} y + h_1, z' = z \quad \text{for } -l < x < l. \quad (1-c)$$

In fact, by applying this transformation, the curved perfect reflecting surface marked in red in Fig. 1 will behave as a conducting plane (denoted by green solid line) for EM wave illuminations. This is similar to the idea of the “carpet cloak” [8–11,19], where an obstacle hidden under a conducting surface would be perceived as a flat conducting sheet. Therefore, after the waves travel through the blue-gridded region with conducting boundaries, they will go on propagating as if they just pass a rectangular region with reflecting surfaces. This explains why the waves transmitted along the x axis can still be supported even when the value of $h_2 - h_1$ is far less than the free-space wavelength, and all the transmission modes will be restored at the far side of the transformed region.

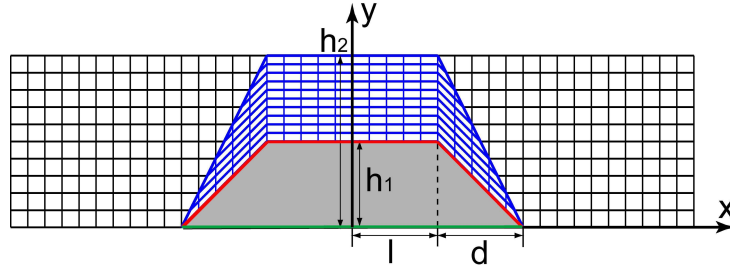


Fig. 1. (a) A coordinate transformation that compresses a trapezoidal region along the y axis into a narrow channel-shaped region.

The corresponding permittivity and permeability tensors of the transformed region can be expressed as [6]:

$$\begin{aligned} \varepsilon_{zz} = \mu_{zz} = \frac{h_2}{h_2 - h_1}, \quad \varepsilon_{xx} = \mu_{xx} = \frac{h_2}{h_2 - h_1}, \quad \varepsilon_{yy} = \mu_{yy} = \frac{h_2 - h_1}{h_2} + \frac{h_2}{h_2 - h_1} \left(\frac{h_1}{d} \right)^2, \\ \varepsilon_{xy} = \mu_{xy} = \frac{h_1 h_2}{(h_2 - h_1) d} \quad \text{for } -(l + d) < x < -l, \end{aligned} \quad (2-a)$$

$$\begin{aligned} \varepsilon_{zz} = \mu_{zz} = \frac{h_2}{h_2 - h_1}, \quad \varepsilon_{xx} = \mu_{xx} = \frac{h_2}{h_2 - h_1}, \quad \varepsilon_{yy} = \mu_{yy} = \frac{h_2 - h_1}{h_2} + \frac{h_2}{h_2 - h_1} \left(\frac{h_1}{d} \right)^2, \\ \varepsilon_{xy} = \mu_{xy} = -\frac{h_1 h_2}{(h_2 - h_1) d} \quad \text{for } l < x < (l + d), \end{aligned} \quad (2-b)$$

$$\varepsilon_{zz} = \mu_{zz} = \frac{h_2}{h_2 - h_1}, \quad \varepsilon_{xx} = \mu_{xx} = \frac{h_2}{h_2 - h_1}, \quad \varepsilon_{yy} = \mu_{yy} = \frac{h_2 - h_1}{h_2} \quad \text{for } -l < x < l. \quad (2-c)$$

$\bar{\bar{\varepsilon}}, \bar{\bar{\mu}}$ appear on the same footing because of the symmetry between electric and magnetic fields. The parameter tensors with elements in Eqs. (2-a) and (2-b) can be easily diagonalized by rotating the coordinate system, and can be realized with uniaxial materials. After the diagonalization process, all the components of the material parameters become positive constants. In addition, by compressing the space in the central tunneling region along the x axis by a factor of $(h_2 - h_1)/h_2$ [20], we can further obtain a set of even simpler parameters:

$$\varepsilon_{zz} = \mu_{zz} = \left(\frac{h_2}{h_2 - h_1} \right)^2, \varepsilon_{xx} = \mu_{xx} = 1, \varepsilon_{yy} = \mu_{yy} = 1 \quad \text{for } -l < x < -l. \quad (3)$$

For a certain TE (or TM) incidence, only ε_{zz} , μ_{xx} , μ_{yy} (or μ_{zz} , ε_{xx} , ε_{yy}) enter into the Maxwell's equations, so the material in the channel can be replaced by the isotropic material with parameters $\varepsilon = [h_2/(h_2 - h_1)]^2$, $\mu = 1$ (or $\varepsilon = 1$, $\mu = [h_2/(h_2 - h_1)]^2$), while still allowing for the tunneling effect.

3. Numerical simulations

The COMSOL Multiphysics finite element-based electromagnetics solver is used to verify the above idea and illustrate the wave propagation. In all the simulations, the working frequency of the EM waves is for simplicity 2GHz.

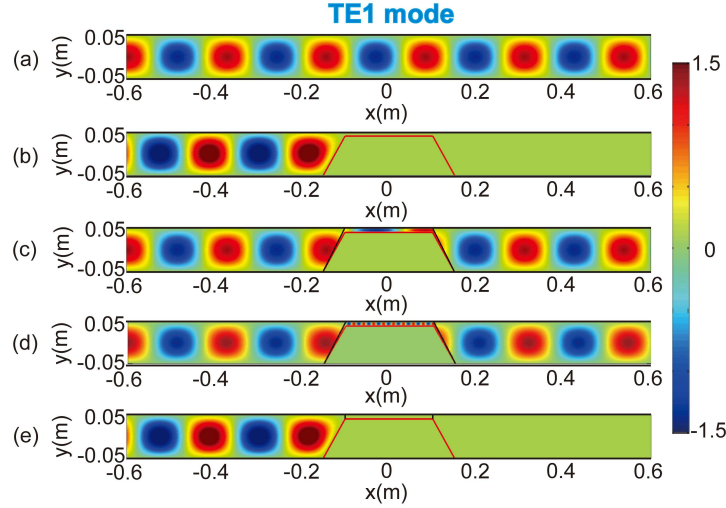


Fig. 2. The electric field distribution inside planar waveguides under the TE1 mode excitation in five different conditions. In all the cases, the width of the planar waveguide is 0.1m, while the width of the channel is 0.01m. (a) an empty waveguide (b) a sub-wavelength channel is embedded inside the waveguide. (c) a waveguide with the embedded channel filled with metamaterial substrate characterized by Eq. (2). (d) a waveguide with the same matched layers at both sides with those in (c), but high- ε isotropic material in the central channel (e) a waveguide with only high- ε isotropic material in the central channel.

We first consider a case where only the first transverse-electric (TE1) mode is supported by a 0.1m ($h_2 = 0.1$ m) wide planar waveguide, as shown in Fig. 2(a). When a channel with a width of only 0.02m ($h_2 - h_1 = 0.01$ m) is embedded in the waveguide (the boundaries of the channel is PEC, and $l = 0.1$ m, $d = 0.05$ m for all the following simulations), the EM wave is cut off and the fields vanish in the wedgy junction region between the waveguide and the channel, as depicted by Fig. 2(b). Next we fill the narrow channel as well as the junction regions on the two sides with the materials characterized by Eq. (2). The result in Fig. 2(c) shows that all the fields are guided through the narrow channel to the planar waveguide on the other side. Compared with the electric field distributions shown in Fig. 2(a), we see that the EM waves continue propagating after passing the channel as if they have never met any blockage in the original waveguide. In Fig. 2(d), all the simulation setups are the same as those in Fig. 2(c) except that we use the material characterized by Eq. (3) for the channel, and similar results could be observed. To understand the tunneling effect from a physical perspective, we may need to check the dispersion relation $k_x^2 + k_y^2 = \omega^2 \varepsilon_r \mu_r$, $\mu = 1$, $k_y = m\pi/d$, $m = 1, 2, 3, \dots$ where d is the width of the planar waveguide. When ε_r increases to a value large enough, k_x will change from imaginary to real. In other words, the cut-off frequency for the waveguide will

decrease if it is stuffed with materials with high permittivity or permeability. In the simulation shown in Fig. 2(d), the material in the channel has an ε equal to 100, so the propagation of waves would still be supported even though the width of the channel is only 1/15 of the free-space wavelength.

Next, we address whether we can achieve the tunneling effect by simply inserting high-permittivity medium in the channel. Figure 2(e) gives the answer, showing that the EM wave still cannot be transmitted in this case. This is because the impedance is strongly mismatched at the junction of waveguides with different cross-sections. In fact, the layers on both sides of the channel $(-l + d) < x < -l$ and $l < x < (l + d)$ are used for the purpose of impedance matching. The performance of the matching layers can be studied theoretically as follows.

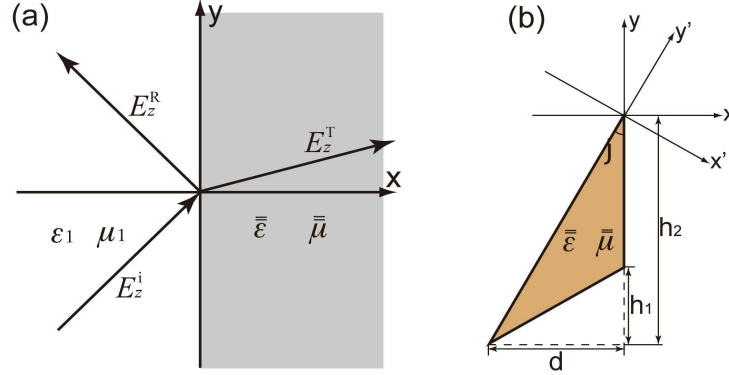


Fig. 3. Schematic figure showing (a) an EM wave with electric field polarized along z direction is incident upon the interface of an isotropic (left) and an anisotropic media (right). (b) a triangular anisotropic medium with free space on the left side, high- ε medium on the right side.

For TE polarized incident waves (where the electric field is polarized along z direction $\vec{E} = \hat{z}E_z$), the wave equation in a medium characterized by $\bar{\varepsilon}$ and $\bar{\mu}$ can be written as

$$\mu_{yy} \frac{\partial^2}{\partial y^2} E_z + \mu_{xx} \frac{\partial^2}{\partial x^2} E_z + (\mu_{xy} + \mu_{yx}) \frac{\partial^2}{\partial x \partial y} E_z + \varepsilon_{zz} (\mu_{xx} \mu_{yy} - \mu_{xy} \mu_{yx}) k_0^2 E_z = 0. \quad (4)$$

The general solution to the above equation is $E_z = A e^{ik_x x + ik_y y}$, where A is an arbitrary constant and $\mu_{xx} k_x^2 + \mu_{yy} k_y^2 + (\mu_{xy} + \mu_{yx}) k_x k_y = \varepsilon_{zz} (\mu_{xx} \mu_{yy} - \mu_{xy} \mu_{yx}) k_0^2$. To simplify our calculation, we first suppose there is no reflection from the interface between the air and the matching layer and only consider the interface between the matching layer and the channel. Then the problem will be simplified to the case shown in Fig. 3(a) where a TE wave is incident upon the interface between an isotropic medium and an anisotropic medium. The incident electric field, reflected electric field and the transmitted electric field can be expressed as

$$\vec{E}^i = \hat{z} E_0 e^{ik_x x + ik_y y}, \vec{E}^R = \hat{z} R E_0 e^{-ik_x x + ik_y y}, \vec{E}^T = \hat{z} T E_0 e^{k'_x x + ik_y y}. \quad (5)$$

By applying the continuity of E_z and H_y at $x = 0$, we obtain

$$1 + R = T, \text{ and } 1 - R = \frac{\mu_1 (\mu_{yx} k_y + \mu_{xx} k'_x)}{k_x (\mu_{xx} \mu_{yy} - \mu_{xy} \mu_{yx})} T. \quad (6)$$

Since the permeability tensor in our case is symmetric ($\mu_{xy} = \mu_{yx}$), we have $k_x = \sqrt{\mu_1 \varepsilon_1 k_0^2 - k_y^2}$ and $k'_x = \left[-\mu_{xy} k_y + \sqrt{(\mu_{xx} \mu_{yy} - \mu_{xy}^2) (\mu_{xx} \varepsilon_{zz} k_0^2 - k_y^2)} \right] / \mu_{xx}$. From Eq. (6) we know that the total transmission condition ($R = 0, T = 1$) requires:

$$\mu_1 \sqrt{\frac{\mu_{xx} \varepsilon_{zz} k_0^2 - k_y^2}{(\mu_1 \varepsilon_1 k_0^2 - k_y^2)(\mu_{xx} \mu_{yy} - \mu_{xy}^2)}} = 1. \quad (7)$$

It is easy to find that as long as $\mu_{xx} \varepsilon_{zz} = \mu_1 \varepsilon_1$, Eq. (7) is independent of k_y and can be further simplified to $\mu_{xx} \mu_{yy} - \mu_{xy}^2 = \mu_1^2$. Since in the case we discussed the condition $\mu_1 = 1$ holds, the total transmission condition can now be summarized as

$$\mu_{xx} \mu_{yy} - \mu_{xy}^2 = 1, \quad \mu_{xx} \varepsilon_{zz} = \varepsilon_1. \quad (8)$$

The fact that Eq. (8) is satisfied for material parameters of the matching layer at the interface implies there is no wave reflection at this interface. The interface between the matching layer and the air can be similarly treated except that we need to rotate the coordinate system (x, y, z) by an angle $-\varphi$ to a new system (x', y', z') , where $\varphi = \arctan(d/h_2)$ [see Fig. 3(b)]. Similarly, the fact that $\mu'_{xx} \mu'_{yy} - \mu'^2_{xy} = 1$, $\mu'_{xx} \varepsilon_{zz} = 1$ means the waves will not reflect at the interface between the matching layer and the free space.

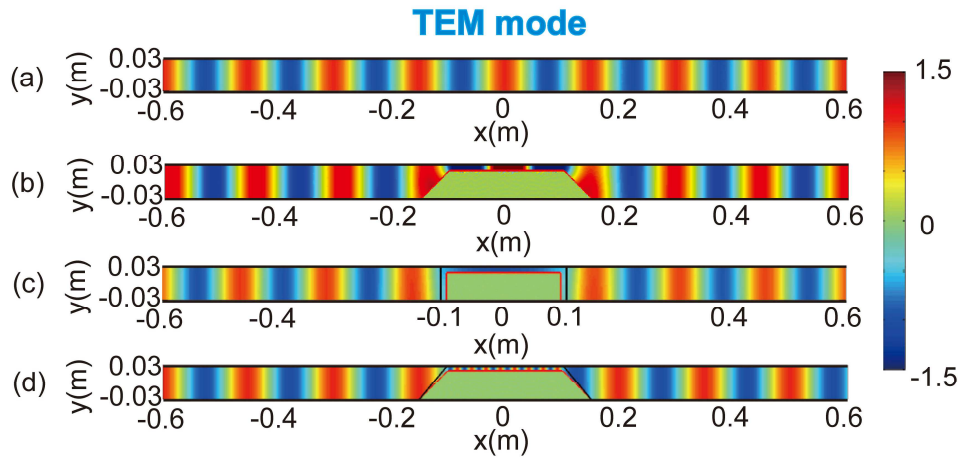


Fig. 4. The electric field distribution for planar waveguides under TEM mode excitation in four different conditions. In all the cases, the widths of the planar waveguide and the channel are 0.06m and 0.01m, respectively. (a) an empty waveguide. (b) a sub-wavelength channel is embedded in the waveguide. (c) the sub-wavelength channel is filled with ENZ metamaterial [2]. (d) the sub-wavelength channel is filled with transformation media characterized by Eqs. (2-a), (2-b), and (3).

From the above analysis, we can see that our proposed approach is able to realize TE mode wave tunneling. In fact, for the TEM wave case, which can be dealt with using ENZ material, our approach is also applicable. Figure 4(a) is the electric field distribution for a waveguide supporting TEM wave, and Fig. 4(b) shows the electric field distribution when a narrow channel is built inside the waveguides. Here $h_2 = 0.06\text{m}$, $h_1 = 0.05\text{m}$. The reflection introduced by the discontinuity of the boundary can be observed by comparing the magnitude of electric fields on the two sides of the channel. In Fig. 4(c) and 4(d), we simulate the tunneling performance of the ENZ material method and our transformation method, respectively. However, different with the ENZ material method which can only be applied to TEM polarization cases, our approach can also treat EM waves in higher modes.

In the following simulation, we increase the width of the planar waveguides to 0.3m so that TM3 mode can be supported. In order to observe the high mode clearly, we only let TM3 mode be excited in our simulation, as displayed in Fig. 5(a). When a 0.02m wide channel is constructed inside such a planar waveguide, the TM3 mode will be cut off at the narrow channel and some lower mode will be excited at the discontinuous boundary. Therefore, some energy is transmitted through the channel without maintaining the original mode, which can be observed in Fig. 5(b). After we fill the channel with high- ε material as well as adding the

two matching layers, the waves can be guided smoothly through the channel and continue propagating in the planar waveguide with TM3 mode [see Fig. 5(c)]. As a comparison, we also simulate how the ENZ material scheme would operate in this case. The result in Fig. 5(d) demonstrates that the ENZ material cannot achieve the tunneling effect for the waves of higher modes.

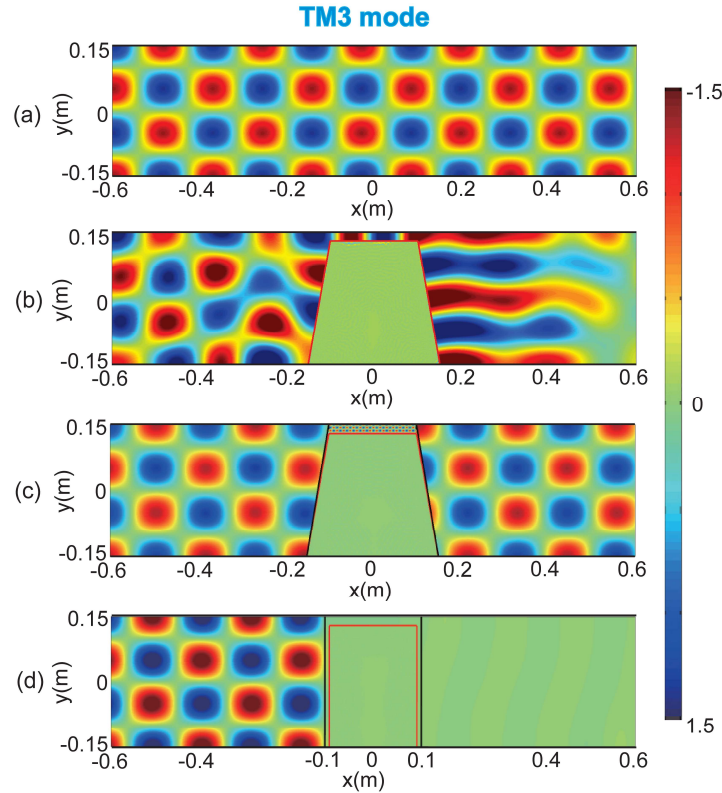


Fig. 5. The electric field distribution for planar waveguides under TM3 mode excitation in five different conditions. In all the cases, the widths of the waveguide and the channel are 0.3m and 0.02m, respectively. (a) an empty waveguide. (b) a narrow channel is embedded in the waveguide. (c) the sub-wavelength channel is filled with transformation media characterized by Eqs. (2-a), (2-b), and (3). (d) the sub-wavelength channel is filled with ENZ metamaterial.

4. Conclusion

A new method which allows for the EM wave squeezing and transmission through narrow channels and waveguide junctions, was introduced in this paper. The validity of this approach was theoretically analyzed and numerically tested in different polarization conditions, using waveguides with different cross sections. Instead of being cut off, all the energies can be totally tunneled through the narrow channel independent of the polarization of the incoming waves. The information of transmission modes will also be persevered after the tunneling operation. Transformation optics provides the theoretical background of this scheme and uniaxial metamaterials provide the means of achieving the prescribed parameters.

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